Heritage High School

Summer Packet

PAP Geometry

The Algebra skills that you will use the most in Pre-AP Geometry at HHS are:

- Solving Linear Equations
- Solving Systems of Linear Equations
- Solving Quadratic Equations (usually by factoring)

You will benefit from reviewing those skills prior to the beginning of school in August. This packet will not be taken for a grade – it is just for you to practice the Algebra skills you should already know!

We look forward to having you in class next year. Enjoy your summer!

Order of Operations

PEMDAS = **P**arentheses, **E**xponents, **M**ultiplication/**D**ivision, **A**dd/**S**ubtract from left to right.

A. Simplify each expression using appropriate Order of Operations.

1. $1 \bullet 5 - 6 \div 2 + 3^2$
2. $4 + 2(10 - 4 \bullet 6)$
3. $3(2+7)^2 \div 5$
4. $3^2 \div 3 + 2^2 \bullet 7 - 20 \div 5$

Solving Equations

The five steps to solving an equation are:

- ✓ Get rid of parentheses
- ✓ Simplify the left side and the right side of the equation as much as possible, i.e. combine any and all like terms
- ✓ Get the variable term on just one side
- ✓ Get the variable term by itself
- ✓ Solve for the variable.

B. Solve for the variable in each problem.

5. $5(3x-2) = 35$
6. $\frac{1}{3}(6x+24) - 20 = -\frac{1}{4}(12x-72)$
7. $5r - 2(2r+8) = 16$
8. $13 - (2c+2) = 2(c+2) + 3c$
9. $\frac{1}{4}(8y+4) - 17 = -\frac{1}{2}(4y-8)$
10. $12 - 3(x-5) = 21$

Solving Inequalities

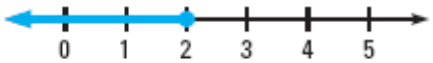
Symbol	Meaning	Equation or Inequality	Graph
=	equals	$x = 3$	
<	is less than	$x < 3$	
≤	is less than or equal to	$x \le 3$	
>	is greater than	$x > 3$	
≥	is greater than or equal to	$x \ge 3$	

Examples:

$2x + 1 \le 5$

$2x \le 4$ Subtract 1 from each side

$x \le 2$ Divide each side by 2



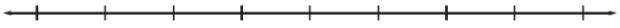
$-4y < 18$

$\frac{-4y}{-4} > \frac{18}{-4}$ Divide by -4 and change < to >

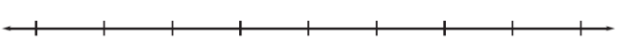
$y > -4.5$ Simplify

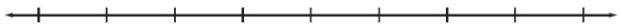
C. Solve and graph the following inequalities.

11. $3f - 4 < 2f + 5$



12. $5(1-x) \ge 4(3-x)$



13. $2d-5<-7 \text{ or } 7<2d-5$ 

14. $-1\le 3-5m\le 3$ 

Graphs and Equations of Lines

Slope-Intercept Form

y = mx + b, where m = slope and b = y-intercept

Graphing Equations in Slope-Intercept Form

1. Write the equation in slope-intercept form for y
2. Find the y-intercept and use it to plot the point where the line crosses the y-axis.
3. Find the slope and use it to plot a second point on the line.
4. Draw a line through the two points.

Writing the Equation: Given the Slope and a y - intercept

Example:

Write an equation of the line that passes through (0, 4) and has a slope of -5. (These can also be given on a graph)

Step 1: Substitute – 5 for m. y = -5x + b

Step 2: Substitute 4 for b (since it is the y –intercept) y = -5x + 4

Point-Slope Form

$y - y_1 = m(x - x_1)$ where m = slope and (x_1, y_1) is the point.

Graphing Equations in Slope-Intercept Form

1. Plot the point (x_1, y_1).
2. Find the slope and use it to plot a second point on the line.
3. Draw a line through the two points.

Writing the Equation: Given a point and a slope

Example:

Write an equation of the line that passes through the point (2, 5) and has a slope of 4. (These can also taken from a graph)

Substitute 2 for x_1, 5 for y_1, and 4 for x y – 5 = 4(x -2)

Given Two Points

Step 1: Find the slope of the line using the two points and the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Step 2: Choose either point and follow the steps above depending on the form you are asked to use.

Standard Form

ax + by = c where a is a *positive*, and a and b are *whole* numbers.

Writing the Equation:

Write the equation using slope-intercept or point-slope form, then rearrange to standard form.

Example:

Write the equation of the line that passes through the point (4, 5) and has a slope of $\frac{1}{2}$.

Step 1: Write in Point-Slope Form y – 5 = $\frac{1}{2}$ (x – 4)

Step 2: Distribute y – 5 = $\frac{1}{2}$ x – 3

Step 3: Subtract $\frac{1}{2}$ x and add 5 - $\frac{1}{2}$ x + y = 2

Step 4: Multiply by -2 to make *a* a positive, whole number x – 2y = -4

D. Find the slope of the line containing each pair of points.

18. (5, 0) and (6, 8)

19. (-2, -4) and (-9, -7)

E. Find the slope of each line

20. y = 7

21. x = -4

22. 2x + y = 15

23. x – 2y = 7

F. Find the equation of the line with the given slope through the given point. Write the answer in *slope-intercept form*.

24. m = -2; (4, 7)

25. $m = -\frac{4}{3}$; (3, -1)

G. Find the equation of the line containing the following points. Write answer in *standard form*.

26. (2, 6) and (4, 1)

27. (-2, -3) and (-4, -6)

H. Write the equation of the line.

28. The horizontal line passing through (2, 5)

29. The vertical line containing the point (-5, 3).

I. Write the equation of the line in slope-intercept form.

30. The line containing (3, 1) and (4, 8)

31. The line containing (3, 3) and (-6, 9)

32. The line with slope $\frac{4}{5}$ and containing (-1, 7)

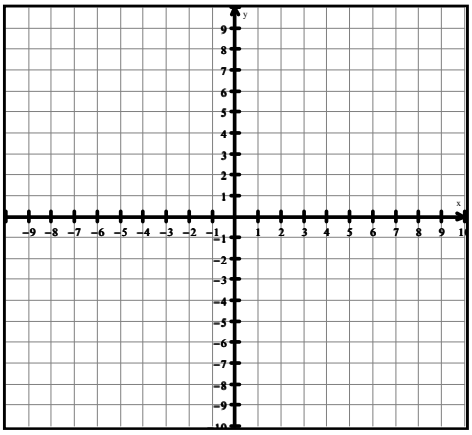
Graph the following equations. Graph three points and label the line with its equation.

K. Point-Slope Form

33. y – 3 = 2(x – 1)

34. $y - 5 = \frac{2}{3}(x - 2)$

33.



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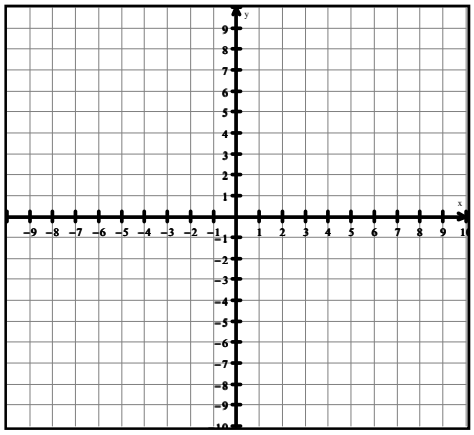

L. Slope-Intercept Form

35. $y = -\frac{1}{2}x - 5$

36. y = -2x + 3

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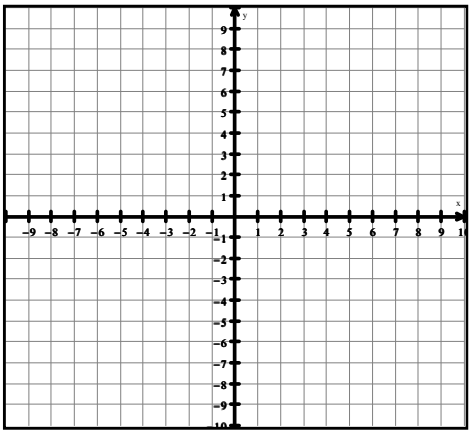

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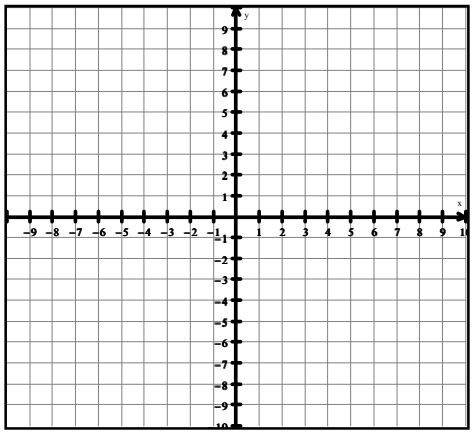
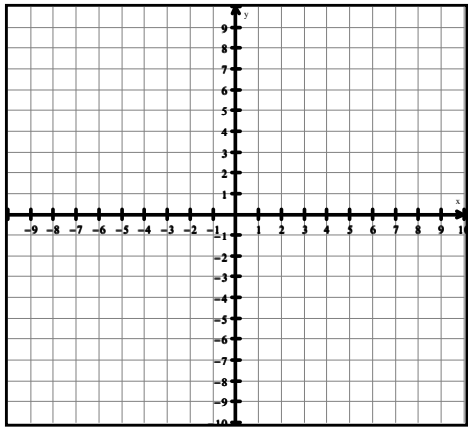
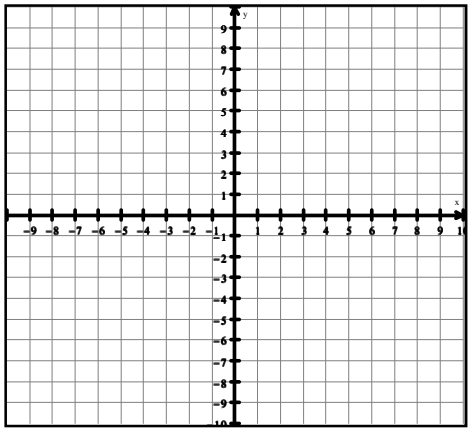


M. Standard Form

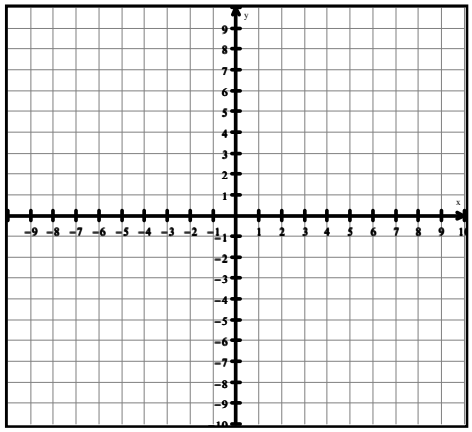
37. y = 3



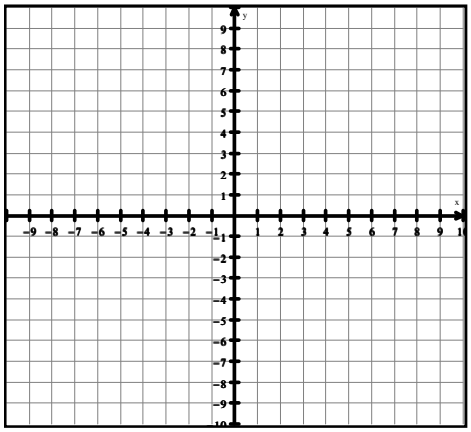
38. 2x – 3y = 12



39. 2x = 4



40. x + 2y = 8



Systems of Linear Equations

Substitution Method

Solve: $\begin{cases} y = 5 - 2x \\ 5x - 6y = 21 \end{cases}$

Solution: Substitute 5 – 2x for y.

5x – 6(5 – 2x) = 21
5x – 30 + 12x = 21
17x – 30 = 21
17x = 51
x = 3

Then substitute 3 for x: y = 5 – 2(3)
y = -1

Answer: (3, -1)

Elimination Method

Example 1 - Solve: $\begin{cases} 3x + 4y = 9 \\ -3x - 2y = -3 \end{cases}$

Solution: 3x + 4y = 9
(+) -3x – 2y = -3
ADD -2y = 6
y = -3

Then substitute -3 for y: 3x + 4(-3) = 9
3x = 21
x = 7

Answer: (7, -3)

Example 2 – Solve: $\begin{cases} 5x - 2y = -19 \\ 2x + 3y = 0 \end{cases}$

Solution: 3(5x – 2y = -19)
2(2x + 3y = x)
→ 15x – 6y = -57
(+) 4x + 6y = 0
ADD 19x = -57
x = -3

Then substitute -3 for x: 2(-3) + 3y = 0
3y = 6
x = 2

Answer: (-3, 2)

N. Solve

41. $\begin{cases} x = 3y - 4 \\ 2x - y = 7 \end{cases}$

42. $\begin{cases} 3y + 2x = 2 \\ -2y + x = 8 \end{cases}$

43. $\begin{cases} x - 2y = 0 \\ 4x - 3y = 15 \end{cases}$

44. $\begin{cases} y-2x=0 \\ 3x+7y=17 \end{cases}$

45. $\begin{cases} 8x+4y=6 \\ 4x=3-y \end{cases}$

46. $\begin{cases} 3x-4y=16 \\ 5x+6y=14 \end{cases}$

47. $\begin{cases} 3x-2y=10 \\ 5x+3y=4 \end{cases}$

48. $\begin{cases} 2x+5y=9 \\ 3x-2y=4 \end{cases}$

49. $\begin{cases} 3x-8y=11 \\ x+6y=8 \end{cases}$

Exponents

$a^0=1$ **Example:** $5^0=1$

$a^m \bullet a^n = a^{m+n}$ **Example:** $x^2 \bullet x^4 = x^{2+4} = x^6$

$\frac{a^m}{a^n} = a^{m-n}$ **Example:** $\frac{b^7}{b^3} = b^{7-3} = b^4$

$(a^m)^n = a^{m(n)}$ **Example:** $(y^3)^4 = y^{3(4)} = y^{12}$

$a^{-m} = \frac{1}{a^m}$ **Example:** $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

O. Simplify (no negative exponents)

50. $\left(\frac{2}{3}\right)^{-2}$

51. $\left(\frac{5}{3}\right)^{-3}$

52. $x^{-1} \bullet x^{-2}$

53. $\left(x^2\right)^{-2}$

54. $\left(5a^2b^3\right)\left(a^{-2}b\right)$

55. $\left(-2ab^5\right)\left(-4ab^{-3}\right)$

56. $x^3 \bullet x^6$

57. $\left(2a^4\right)\left(5a^3\right)$

58. $\left(a^2\right)^3$

59. $\left(5a\right)^2$

60. $c \bullet c^5 \bullet c^2$

61. $\left(-2xy^2\right)\left(-3x^2y\right)$

Multiplying Binomials

$$(2x-4)(3x+5) = \underset{\text{First terms}}{6x^2} + \underset{\text{Outer terms}}{10x} - \underset{\text{Inner terms}}{12x} - \underset{\text{last terms}}{20} = \underbrace{6x^2-2x-20}_{\text{combine like terms}}$$

$$(3x-4)^2 = (3x-4)(3x-4) = \underset{\text{First terms}}{9x^2} - \underset{\text{Outer terms}}{12x} - \underset{\text{Inner terms}}{12x} + \underset{\text{last terms}}{16} = \underbrace{9x^2-24x+16}_{\text{combine like terms}}$$

P. Multiply the following binomials.

62. $(x+3)(x+4)$

63. $(2x+1)(x+4)$

64. $(6x+5)(2x-1)$

65. $(x-4)(x+4)$

66. $(x-6)^2$

67. $(6x+5y)^2$

Factoring Polynomials.

Examples:

1) **EX:** $a^2-16=(a+4)(a-4)$; $25a^2-36x^6=(5a+6x^3)(5a-6x^3)$ **- PATTERN DIFFERENCE OF SQUARES**

2)

All others

1)	**ALWAYS look for a GCF first.**	**$10x^3 - 24x^2 - 18x$**	**GCF = 2x**
2)	**Multiply the coefficients of the first and third terms.**	**$2x(5x^2 - 12x - 9)$**	**5 * - 9 = -45 (-15, 3)**
3)	**Find the factors whose sum is equal to the center term.**		
4)	**Rewrite as a polynomial with 4 terms.**	**$2x(5x^2 - 15x + 3x - 9)$**	
5)	**Group the first two terms and factor out GCF**	**$2x[5x(x - 3) + 3(x - 3)]$**	
	Repeat with last two terms		
6)	**Write your binomials (don't forget GCF from Step 1)**	**$2x (x - 3)(5x + 3)$**	

Q. Factor each of the following polynomials.

68. $10x^2-19x+6$

69. x^2-81

70. $3x^2+13x+4$

71. $9x^4-25y^{10}$

72. $x^2-7x+12$

73. $10x^3+45x^2+20x$

Solving Quadratic Equations

Solve using Square Roots

Problem:	$5x^2-75=0$
Get numbers on one side of equation	$\frac{5x^2}{5}=\frac{75}{5}$
Divide by 5	$x^2=15$
Square root both sides	$x=\pm\sqrt{15}$

Problem	$(x+6)^2=21$
Square root both sides	$\sqrt{(x+6)^2}=\pm\sqrt{21}$
Square root of $\sqrt{(x+6)^2}=(x+6)$	$x+6=\pm\sqrt{21}$
subtract 6 from both sides	$-6 \quad -6$
Answer:	$x=\pm\sqrt{21}-6$

Solve using Factoring

Problem	$a^2+12a-45=0$	
Factor the problem	$(a+15)(a-3)=0$	
Make each factor equal to zero and solve for "x"	$a+15=0$ *and*	$a-3=0$
	$-15 \quad -15$	$+3 \quad +3$
Answer	a = -15	a = 3

R. Solve each quadratic equation using square roots.

74. $x^2=121$

75. $3x^2=30$

76. $4x^2-25=0$

77. $(x-2)^2=49$

78. $(b-3)^2=6$

79. $(y+4)^2=36$

Solve each quadratic equation using factoring.

80. $x^2+7x=0$

81. $p^2-16p+48=0$

82. $x^2+7x+6=0$

83. $m^2+4m=21$

84. $t^2=9t-14$

85. $2x^2+12x=-10$

Simplifying Radicals

EX: Write in Simplest Radical Form: $\sqrt{252}=\sqrt{3\bullet 3\bullet 2\bullet 2\bullet 7}=3\bullet 2\bullet\sqrt{7}=6\sqrt{7}$

Factor to prime factors then look for pairs

S. Simplify the following radicals

86. $\sqrt{18}=$

87. $\sqrt{24}=$

88. $\sqrt{32}=$

89. $\sqrt{40}=$

90. $\sqrt{363}=$

91. $\sqrt{162}=$

92. $\sqrt{75}=$

93. $\sqrt{147}=$

94. $\sqrt{54}=$

Pythagorean Theorem

Pythagorean Theorem: $a^2+b^2=c^2$**, a and b are the legs and C is the hypotenuse (longest side).**

Examples:

a = 3, b = 6, c = ?		a = 4, b = ?, c = 12
$a^2+b^2=c^2$	Pythagorean Theorem	$a^2+b^2=c^2$
$3^2+6^2=c^2$	Plug in values	$4^2+b^2=12^2$
$9+36=c^2$	Square numbers	$16+b^2=144$
$45=c^2$	Combine numbers/get all numbers on one side	$b^2=120$
$\sqrt{45}=\sqrt{c^2}$	Square root both sides	$\sqrt{b^2}=\sqrt{120}$
$3\sqrt{5}=c$	answer	$2\sqrt{30}$

T. Use Pythagorean Theorem to find the missing side of the right triangles. If c is the measure of the hypotenuse of a right triangle, find each missing measure. Leave all answers in simplest radical form. (See Simplifying Radicals on the previous page)

95. a = 5, b = 12, c = ?

96. a = 8, b = ?, c = 17

97. a = 5, b = ? c = 10

98. a = 5, b = 8, c = ?

99. a = ?, b = 6, c = 14

100. a = $\sqrt{7}$, b = $\sqrt{9}$, c = ?

Area

Triangle:	$A = \frac{1}{2}bh$ where b is the length of the base and h is the height of the triangle.
Square:	$A = s^2$
Parallelogram/Rectangle:	$A = bh$ where b is the length of the base and h is the height
Trapezoid:	$A = \frac{1}{2}h(b_1 + b_2)$ where h is the height, and b_1 and b_2 are the bases
Circle:	$A = \pi r^2$ where r is the radius of the circle

U. Find the area of the following figures. Round to the nearest hundredth if necessary.

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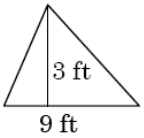

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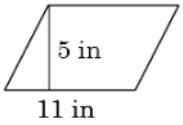

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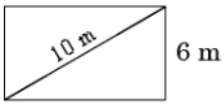

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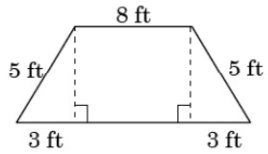

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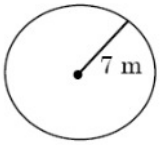

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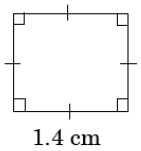

Surface Area and Volume

Prism:	$Surface\ Area = Ph + 2B; Volume = Bh$ where P = Perimeter of base, h = height of prism B = Area of base
Cylinder:	$Surface\ Area = 2\pi rh + 2\pi r^2; Volume = \pi r^2 h$ where r = radius of cylinder h = height of cylinder
Pyramid:	$Surface\ Area = \frac{1}{2}P\ell + B; Volume = \frac{1}{3}Bh$ where P = Perimeter of base, ℓ =slant height, h = height of pyramid B = Area of base
Cone:	$Surface\ Area = \pi r\ell + \pi r^2; Volume = \frac{1}{3}\pi r^2 h$ where r = radius of cone, h = height of cone, ℓ =slant height

V. Find the surface area and volume of the following figures. Round to the nearest hundredth if necessary.

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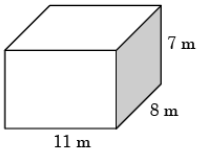

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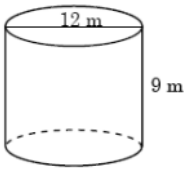

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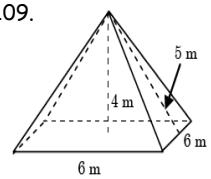

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